Supporting Information for:
“Memetic Viability Evolution for Constrained Optimization”
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1 Introduction

As we described in the main text, the proposed algorithm mViE was designed explicitly for handling problems with inequality constraints only. However, several real-world problems are characterized by the presence of one or more equality constraints, i.e. constraints of the form $h_k(x) = 0$, $k = 1, \ldots, p$, where $x \in \mathbb{R}^n$ is a candidate solution to the problem at hand.

The reason for focusing our research on problems with inequalities only was threefold. The first reason, is purely algorithmic: the CMA-ES structure is indeed not feasible when the volume of the feasible region reduces to zero (which is the case of equality constraints). We discuss this aspect more in detail in Section 3. Secondly, specific techniques for handling equality constraints in evolutionary algorithms have been presented in the literature, such as gradient-mutations [1], quadratic approximations [2, 3] or projection methods [4]. If the mathematical formulation of the constraint is available, one could even try to repair the unfeasible solutions by moving them back onto the constraints [5, 6], or apply a traditional non-linear programming technique as in [7]. In any case, since these techniques can be activated only when the problem presents equality constraints, adding them to our method would be relatively easy and, most of all, would not impair the performances on problems with inequalities only. Finally, narrowing our analysis to problems characterized by inequalities only allowed us to better understand the performance of mViE without the need of introducing an additional operator specific for equalities.

Nevertheless, to provide a baseline of mViE’s performance also on this class of problems (without any specific equality-handling mechanism), we present here additional results obtained on the CEC 2006 benchmark problems with equalities [8]. For simplicity, equality constraints are transformed into inequalities, by introducing a tolerance $\delta$, such that they can be rewritten as $|h_k(x)| - \delta \leq 0$. We set $\delta$ equal to the desired accuracy level $10^{-4}$ defined as in the CEC benchmark specifications. We repeated our method 25 times on each benchmark problem.
In each run, mViE was allowed 500,000 function evaluations. The numerical results are reported in the following section.

## 2 Results

Table 1 reports for each problem the median number of fitness evaluations (NFES) needed by mViE to reach the desired accuracy level, and the corresponding success rate (SR). mViE could solve at least in one run 5 out of the 9 tested problems with equality constraints of the CEC 2006 benchmark set. In four cases (g03, g11, g13 and g15) the success rate was about 100%, a rather impressive result considering that the method does not use any specific mechanism for handling equalities. However, in the g05 problem the algorithm had difficulties in discovering the optimum and failed in many runs, while in the remaining four cases it could not solve the problem even once. This result clearly indicates that, in order to handle more complex landscapes in presence of equality constraints, specific operators are actually needed.

Table 1: Median NFES to achieve the accuracy level \((f(\vec{x}) - f(\vec{x}^*)) \leq 0.0001\) and Success Rate for the selected CEC 2006 problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>NFES</th>
<th>SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>g03</td>
<td>11442</td>
<td>96%</td>
</tr>
<tr>
<td>g05</td>
<td>90715</td>
<td>12%</td>
</tr>
<tr>
<td>g11</td>
<td>1463</td>
<td>100%</td>
</tr>
<tr>
<td>g13</td>
<td>59821</td>
<td>92%</td>
</tr>
<tr>
<td>g14</td>
<td>-</td>
<td>0%</td>
</tr>
<tr>
<td>g15</td>
<td>13882</td>
<td>100%</td>
</tr>
<tr>
<td>g17</td>
<td>-</td>
<td>0%</td>
</tr>
<tr>
<td>g21</td>
<td>-</td>
<td>0%</td>
</tr>
<tr>
<td>g23</td>
<td>-</td>
<td>0%</td>
</tr>
</tbody>
</table>

## 3 Discussion

Introducing equality constraints can considerably reduce the size of the feasible space down to zero-volume regions. In such conditions, a limitation of our proposed approach, and possibly of any other CMA-ES-based approach, is given by the fact that the multivariate Gaussian distribution underlying the covariance matrix adaptation is not suitable for sampling solutions from such a small space. In these cases, indeed, the covariance matrix becomes ill-conditioned as the distribution degenerates.

In order to handle such situations, apart from including in our method one of the aforementioned equality-handling techniques from the literature, it would be worthwhile considering alternative approaches that are more strictly related to the memetic Viability Evolution concept. We highlight here some possibilities:
1. The algorithm could be made adaptive in such a way to disable the (1+1)-
CMA-ES local search units and continue the search only by using DE
operators; however, as reported in the literature (and as we verified in
preliminary experiments, not reported here) standard DE cannot solve
efficiently problems with equalities, unless specific operators are used.

2. Another option could be to further generalize the viability concept, by
taking advantage of the fact that constraints might be not only relaxed
or narrowed, but even added or deleted. In other words, the algorithm
could handle each equality constraint separately adding or removing it as
evolution proceeds, so to control the viability region and allow the search
to escape from local optima lying on the boundaries of the feasible region.

References

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